On the Theory of Mathematical Forms

Ken Kubota

2015

%0
$= (COMPS\,t\,u\,w\,h\,(COMPS\,u\,v\,w\,g\,f))\,(COMPS\,t\,v\,w\,(COMPS\,t\,u\,v\,h\,g)\,f)$
$\ldots$
$\ldots =_{o\omega\omega}(COMPS_{\backslash 6\backslash 4(\backslash 5\backslash 4)(\backslash 5\backslash 4)\tau\tau\tau} t_\tau u_\tau w_\tau h_{tu}(COMPS_{\backslash 6\backslash 4(\backslash 5\backslash 4)(\backslash 5\backslash 4)\tau\tau\tau} u_\tau v_\tau w_\tau g_{uv} f_{vw}))\ldots$
$\ldots(COMPS_{\backslash 6\backslash 4(\backslash 5\backslash 4)(\backslash 5\backslash 4)\tau\tau\tau} t_\tau v_\tau w_\tau (COMPS_{\backslash 6\backslash 4(\backslash 5\backslash 4)(\backslash 5\backslash 4)\tau\tau\tau} t_\tau u_\tau v_\tau h_{tu} g_{uv}) f_{vw})$

5.1.89 Results for File definitions1.r0.txt

##
Basic Definitions
##
##
Source: [Andrews 2002 (ISBN 1-4020-0763-9), p. 212]
##

Written by Ken Kubota (<mail@kenkubota.de>).
##
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For more information visit: <http://dx.doi.org/10.4444/100.10>
##

Definition of truth
:= T $=_{o\omega\omega} =_\omega =_\omega$
wff 12 : $= = =_o$:= T

Definition of falsehood
:= F $=_{o(oo)(oo)}[\lambda x_o.T_o][\lambda x_o.x_o]$
wff 20 : $= [\lambda x.T]\,[\lambda x.x]_o$:= F

Definition of the universal quantifier (with type abstraction)
:= $\forall$ $[\lambda t_\tau.[\lambda p_{ot}.(=_{o(ot)(ot)}[\lambda x_t.T_o]p_{ot})_o]_{(o(ot))}]$
wff 29 : $[\lambda t.[\lambda p.(= [\lambda x.T]\,p)]]_{o(o\backslash 3)\tau}$:= $\forall$

Definition of the conjunction
:= $\wedge$ $[\lambda x_o.[\lambda y_o.(=_{o\omega\omega}[\lambda g_{ooo}.(g_{ooo}T_oT_o)_o][\lambda g_{ooo}.(g_{ooo}x_oy_o)_o])_o]_{(oo)}]$
wff 47 : $[\lambda x.[\lambda y.(= [\lambda g.(g\,T\,T)]\,[\lambda g.(g\,x\,y)])]]_{ooo}$:= $\wedge$

Definition of the implication
:= $\supset$ $[\lambda x_o.[\lambda y_o.(=_{ooo}x_o(\wedge_{ooo}x_oy_o))_o]_{(oo)}]$
wff 53 : $[\lambda x.[\lambda y.(= x\,(\wedge\,x\,y))]]_{ooo}$:= $\supset$

Definition of the negation
:= $\sim$ $[\lambda a_o.(=_{ooo}F_oa_o)_o]$
wff 57 : $[\lambda a.(= F\,a)]_{oo}$:= $\sim$

Definition of the disjunction
:= $\vee$ $[\lambda a_o.[\lambda b_o.(\sim_{oo}(\wedge_{ooo}(\sim_{oo}a_o)(\sim_{oo}b_o)))_o]_{(oo)}]$
wff 65 : $[\lambda a.[\lambda b.(\sim(\wedge\,(\sim a)\,(\sim b)))]]_{ooo}$:= $\vee$

Definition of the existential quantifier (with type abstraction)
:= $\exists$ $[\lambda t_\tau.[\lambda p_{ot}.(\sim_{oo}(=_{o(ot)(ot)}[\lambda x_t.T_o][\lambda x_t.(\sim_{oo}(p_{ot}x_t))_o]))_o]_{(o(ot))}]$
\# wff 72 : $[\lambda t.[\lambda p.(\sim(=[\lambda x.T]\,[\lambda x.(\sim(p\,x))]))]]_{o(o\backslash 3)\tau}$:= $\exists$

Definition of inequality
:= $\neq$ $[\lambda x_\omega.[\lambda y_\omega.(\sim_{oo}(=_{o\omega\omega}x_\omega y_\omega))_o]_{(o\omega)}]$
\# wff 79 : $[\lambda x.[\lambda y.(\sim(=x\,y))]]_{o\omega\omega}$:= $\neq$

5.1.90 Results for File definitions2.r0.txt

##
Further Definitions
##
##
Source: [Andrews 2002 (ISBN 1-4020-0763-9), pp. 231, 233]
##

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##

<< definitions1.r0.txt

Definition of the subset
:= $\subseteq$ $[\lambda t_\tau.[\lambda x_{ot}.[\lambda y_{ot}.(\forall_{o(o\backslash 3)\tau}t_\tau[\lambda z_t.(\supset_{ooo}(x_{ot}z_t)(y_{ot}z_t))_o])_o]_{(o(ot))}]_{(o(ot)(ot))}]$
\# wff 92 : $[\lambda t.[\lambda x.[\lambda y.(\forall\,t\,[\lambda z.(\supset(x\,z)\,(y\,z))])]]]_{o(o\backslash 4)(o\backslash 3)\tau}$:= $\subseteq$

Definition of the power set
:= $\mathcal{P}$ $[\lambda t_\tau.[\lambda y_{ot}.[\lambda x_{ot}.(\subseteq_{o(o\backslash 4)(o\backslash 3)\tau}t_\tau x_{ot}y_{ot})_o]_{(o(ot))}]_{(o(ot)(ot))}]$
\# wff 103 : $[\lambda t.[\lambda y.[\lambda x.(\subseteq t\,x\,y)]]]_{o(o\backslash 4)(o\backslash 3)\tau}$:= $\mathcal{P}$

Definition of the uniqueness quantifier (with type abstraction)
:= $\exists_1$ $[\lambda t_\tau.[\lambda p_{ot}.(\exists_{o(o\backslash 3)\tau}t_\tau[\lambda y_t.(=_{o(ot)(ot)}p_{ot}(=_{ott}y_t))_o])_o]_{(o(ot))}]$
\# wff 112 : $[\lambda t.[\lambda p.(\exists\,t\,[\lambda y.(=p\,(=y))])]]_{o(o\backslash 3)\tau}$:= $\exists_1$

5.1.91 Results for File definitions3.r0.txt

##
New Definitions
##
##
Source: [Kubota 2015 (doi: 10.4444/100.10)]
##

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##
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##

<< definitions2.r0.txt

Definition of the universal set
:= V $[\lambda x_\omega.T_o]$
wff 113 : $[\lambda x.T]_{o\omega}$:= V

Definition of the empty set
:= $\emptyset$ $[\lambda x_\omega.F_o]$
wff 114 : $[\lambda x.F]_{o\omega}$:= $\emptyset$

Definition of the polymorphic identity relation helper function
:= == $[\lambda t_\tau.[\lambda x_t.[\lambda y_t.(=_{ott}x_t y_t)_o]_{(ot)}]_{(ott)}]$
wff 119 : $[\lambda t.[\lambda x.[\lambda y.(= x\, y)]]]_{o\backslash 3\backslash 2\tau}$:= ==

Definition of the polymorphic non-identity relation helper function
:= != = $[\lambda t_\tau.[\lambda x_t.[\lambda y_t.(\sim_{oo}(=_{ott}x_t y_t))_o]_{(ot)}]_{(ott)}]$
wff 126 : $[\lambda t.[\lambda x.[\lambda y.(\sim (= x\, y))]]]_{o\backslash 3\backslash 2\tau}$:= != =

Definition of the polymorphic descriptor helper function
:= I $[\lambda t_\tau.[\lambda x_{ot}.(\iota_{t(ot)}x_{ot})_t]_{(t(ot))}]$
wff 129 : $[\lambda t.[\lambda x.(\iota\, x)]]_{\backslash 2(o\backslash 3)\tau}$:= I

Definition of exclusive disjunction (logical exclusive "or", XOR)
:= XOR $[\lambda x_o.[\lambda y_o.(\sim_{oo}(=_{ooo}x_o y_o))_o]_{(oo)}]$
wff 135 : $[\lambda x.[\lambda y.(\sim (= x\, y))]]_{ooo}$:= XOR

Definition of commutativity
:= $COMMT$ $[\lambda t_\tau.[\lambda f_{ttt}.(=_{ott}(f_{ttt}x_t y_t)(f_{ttt}y_t x_t))_o]_{(o(ttt))}]$
wff 147 : $[\lambda t.[\lambda f.(= (f\, x\, y)\, (f\, y\, x))]]_{o(\backslash 4\backslash 4\backslash 3)\tau}$:= $COMMT$

Definition of associativity
:= $ASSOC$ $[\lambda t_\tau.[\lambda f_{ttt}.(=_{ott}(f_{ttt}(f_{ttt}x_t y_t)z_t)(f_{ttt}x_t(f_{ttt}y_t z_t)))_o]_{(o(ttt))}]$
wff 159 : $[\lambda t.[\lambda f.(= (f\, (f\, x\, y)\, z)\, (f\, x\, (f\, y\, z)))]]_{o(\backslash 4\backslash 4\backslash 3)\tau}$:= $ASSOC$

5.1.92 Results for File group.r0.txt

##
Groups
##
##
Source: [Kubota 2015 (doi: 10.4444/100.10)]
##

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##
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##

<< basics.r0.txt

.1: Associativity
:= $GrpAss$ ...
$\ldots \forall_{o(o\backslash 3)\tau} g_\tau [\lambda a_g.(\forall_{o(o\backslash 3)\tau} g_\tau [\lambda b_g.(\forall_{o(o\backslash 3)\tau} g_\tau [\lambda c_g.(=_{ogg}(l_{ggg}(l_{ggg} a_g b_g) c_g)(l_{ggg} a_g (l_{ggg} b_g c_g)))_o])_o])_o]$
wff 233 : $\forall g\,[\lambda a.(\forall g\,[\lambda b.(\forall g\,[\lambda c.(=(l\,(l\,a\,b)\,c)\,(l\,a\,(l\,b\,c)))])])]_o$:= $GrpAss$

.2: Identity element
:= $GrpIdy$ $\forall_{o(o\backslash 3)\tau} g_\tau [\lambda a_g.(\wedge_{ooo}(=_{ogg}(l_{ggg} a_g e_g) a_g)(=_{ogg}(l_{ggg} e_g a_g) a_g))_o]$
wff 245 : $\forall g\,[\lambda a.(\wedge\,(=(l\,a\,e)\,a)\,(=(l\,e\,a)\,a))]_o$:= $GrpIdy$

.3: Inverse element
:= $GrpInv$ $\forall_{o(o\backslash 3)\tau} g_\tau [\lambda a_g.(\exists_{o(o\backslash 3)\tau} g_\tau [\lambda b_g.(\wedge_{ooo}(=_{ogg}(l_{ggg} a_g b_g) e_g)(=_{ogg}(l_{ggg} b_g a_g) e_g))_o])_o]$
wff 257 : $\forall g\,[\lambda a.(\exists g\,[\lambda b.(\wedge\,(=(l\,a\,b)\,e)\,(=(l\,b\,a)\,e))])]_o$:= $GrpInv$

##
Definition of group (all three group properties combined)
##

:= Grp $[\lambda g_\tau.[\lambda l_{ggg}.(\wedge_{ooo} GrpAss_o (\exists_{o(o\backslash 3)\tau} g_\tau [\lambda e_g.(\wedge_{ooo} GrpIdy_o GrpInv_o)_o]))_o]_{(o(ggg))}]$
wff 266 : $[\lambda g.[\lambda l.(\wedge\,GrpAss\,(\exists\,g\,[\lambda e.(\wedge\,GrpIdy\,GrpInv)]))]]_{o(\backslash 4\backslash 4\backslash 3)\tau}$:= Grp

Group property identity element only (with identity element abstracted)
:= $GrpIdO$ $[\lambda g_\tau.[\lambda l_{ggg}.[\lambda e_g.GrpIdy_o]_{(og)}]_{(og(ggg))}]$
wff 270 : $[\lambda g.[\lambda l.[\lambda e.GrpIdy]]]_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}$:= $GrpIdO$

5.1.93 Results for File group_identity_element_unique.r0.txt

##
Uniqueness of the Group Identity Element
##
##
Source: [Kubota 2015 (doi: 10.4444/100.10)]
##

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##

<< basics.r0.txt
<< K8005.r0.txt
<< group.r0.txt

shorthands
:= $\$HYPTH$ $\wedge_{ooo}(\wedge_{ooo}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}g_\tau l_{ggg})(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}g_\tau l_{ggg}e_g))\ldots$
$\ldots(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}g_\tau l_{ggg}f_g)$
wff 1446 : $\wedge(\wedge(Grp\,g\,l)\,(GrpIdO\,g\,l\,e))\,(GrpIdO\,g\,l\,f)_o$:= $\$HYPTH$
:= $\$TMPDED$ $\forall_{o(o\backslash 3)\tau}g_\tau[\lambda a_g.(\wedge_{ooo}(=_{ogg}(l_{ggg}a_g f_g)a_g)(=_{ogg}(l_{ggg}f_g a_g)a_g))_o]$
wff 1457 : $\forall g\,[\lambda a.(\wedge(=(l\,a\,f)\,a)\,(=(l\,f\,a)\,a))]_o$:= $\$TMPDED$

.1: Let (g,l) be a group, and e and f identity elements of it

$\%K8005$
$\supset x\,x$:= $K8005$
$\supset_{ooo}x_o x_o$:= $K8005$

use Proof Template A5221 (Sub): B → B [x/A]
:= $\$B5221$ %0
wff 1357 : $\supset x\,x_{o,\ldots}$:= $\$B5221$ $K8005$
:= $\$T5221$ o
wff 2 : o_τ := $\$T5221$
:= $\$X5221$ x_o
wff 16 : x_o := $\$X5221$
:= $\$A5221$ $\wedge_{ooo}(\wedge_{ooo}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}g_\tau l_{ggg})(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}g_\tau l_{ggg}e_g))(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}g_\tau l_{ggg}f_g)$
wff 1446 : $\wedge(\wedge(Grp\,g\,l)\,(GrpIdO\,g\,l\,e))\,(GrpIdO\,g\,l\,f)_o$:= $\$A5221$ $\$HYPTH$
<< A5221.r0t.txt
:= $\$B5221$
:= $\$T5221$
:= $\$X5221$
:= $\$A5221$

:= $\$FULLH$ %0
wff 1494 : $\supset\$HYPTH\,\$HYPTH_{o,\ldots}$:= $\$FULLH$

.2: Proof of H ⊃ e * f = e

$\%\$FULLH$
$\supset\$HYPTH\,\$HYPTH$:= $\$FULLH$
$\supset_{ooo}\$HYPTH_o\$HYPTH_o$:= $\$FULLH$

use Proof Template K8019H: H ⊃ (A ∧ B) → H ⊃ A, H ⊃ B
:= $\$H8019H$ %0
wff 1494 : $\supset\$HYPTH\,\$HYPTH_{o,\ldots}$:= $\$FULLH$ $\$H8019H$
<< K8019H.r0t.txt
:= $\$H8019H$
$\%\$B8019H$
$\supset\$HYPTH\,(GrpIdO\,g\,l\,f)$:= $\$B8019H$
$\supset_{ooo}\$HYPTH_o(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}g_\tau l_{ggg}f_g)$:= $\$B8019H$

:= $\$A8019H$
:= $\$B8019H$
%0
$\supset \$HYPTH\, (GrpIdO\, g\, l\, f)$
$\supset_{ooo}\$HYPTH_o(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}g_\tau l_{ggg}f_g)$

§\ $GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}g_\tau$
$= (GrpIdO\, g)\, [\lambda l.[\lambda e.GrpIdy]]$
§s %1 12 %0
$\supset \$HYPTH\, ([\lambda l.[\lambda e.GrpIdy]]\, l\, f)$
§\ $[\lambda l_{ggg}.[\lambda e_g.GrpIdy_o]_{(og)}]l_{ggg}$
$= ([\lambda l.[\lambda e.GrpIdy]]\, l)\, [\lambda e.GrpIdy]$
§s %1 6 %0
$\supset \$HYPTH\, ([\lambda e.GrpIdy]\, f)$
§\ $[\lambda e_g.GrpIdy_o]f_g$
$= ([\lambda e.GrpIdy]\, f)\, \$TMPDED$
§s %1 3 %0
$\supset \$HYPTH\, \$TMPDED$

use Proof Template A5215H (∀ I): H ⊃ ∀ x: B → H ⊃ B [x/a]
:= $\$T5215H$ g_τ
wff 1371 : g_τ := $\$T5215H$
:= $\$X5215H$ $a_{\$T5215H}$
wff 1375 : $a_{\$T5215H}$:= $\$X5215H$
:= $\$A5215H$ $e_{\$T5215H}$
wff 1397 : $e_{\$T5215H}$:= $\$A5215H$
:= $\$H5215H$ %0
wff 1872 : $\supset \$HYPTH\, \$TMPDED_o$:= $\$H5215H$
<< A5215H.r0t.txt
:= $\$T5215H$
:= $\$X5215H$
:= $\$A5215H$
:= $\$H5215H$
%0
$\supset \$HYPTH\, (\wedge\, (=(l\, e\, f)\, e)\, (=(l\, f\, e)\, e))$
$\supset_{ooo}\$HYPTH_o(\wedge_{ooo}(=_{ogg}(l_{ggg}e_g f_g)e_g)(=_{ogg}(l_{ggg}f_g e_g)e_g))$

use Proof Template K8019H: H ⊃ (A ∧ B) → H ⊃ A, H ⊃ B
:= $\$H8019H$ %0
wff 1943 : $\supset \$HYPTH\, (\wedge\, (=(l\, e\, f)\, e)\, (=(l\, f\, e)\, e))_o$:= $\$H8019H$
<< K8019H.r0t.txt
:= $\$H8019H$
%$\$A8019H$
$\supset \$HYPTH\, (=(l\, e\, f)\, e)$:= $\$A8019H$
$\supset_{ooo}\$HYPTH_o(=_{ogg}(l_{ggg}e_g f_g)e_g)$:= $\$A8019H$
:= $\$A8019H$
:= $\$B8019H$

:= $\$EIDTY$ %0

wff 1984 : $\supset \$HYPTH\, (=(l\, e\, f)\, e)_o$:= $\$EIDTY$

.3: Proof of H ⊃ e * f = f

%$\$FULLH$
$\supset \$HYPTH\, \$HYPTH$:= $\$FULLH$
$\supset_{ooo} \$HYPTH_o \$HYPTH_o$:= $\$FULLH$
:= $\$FULLH$

use Proof Template K8019H: H ⊃ (A ∧ B) → H ⊃ A, H ⊃ B
:= $\$H8019H$ %0
wff 1494 : $\supset \$HYPTH\, \$HYPTH_{o,\dots}$:= $\$H8019H$
<< K8019H.r0t.txt
:= $\$H8019H$
%$\$A8019H$
$\supset \$HYPTH\, (\wedge\, (Grp\, g\, l)\, (GrpIdO\, g\, l\, e))$:= $\$A8019H$
$\supset_{ooo} \$HYPTH_o (\wedge_{ooo} (Grp_{o(\backslash 4\backslash 4\backslash 3)\tau} g_\tau l_{ggg}) (GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau} g_\tau l_{ggg} e_g))$:=
$\$A8019H$
:= $\$A8019H$
:= $\$B8019H$

use Proof Template K8019H: H ⊃ (A ∧ B) → H ⊃ A, H ⊃ B
:= $\$H8019H$ %0
wff 1788 : $\supset \$HYPTH\, (\wedge\, (Grp\, g\, l)\, (GrpIdO\, g\, l\, e))_o$:= $\$H8019H$
<< K8019H.r0t.txt
:= $\$H8019H$
%$\$B8019H$
$\supset \$HYPTH\, (GrpIdO\, g\, l\, e)$:= $\$B8019H$
$\supset_{ooo} \$HYPTH_o (GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau} g_\tau l_{ggg} e_g)$:= $\$B8019H$
:= $\$A8019H$
:= $\$B8019H$

§\ $GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau} g_\tau$
$= (GrpIdO\, g)\, [\lambda l.[\lambda e.GrpIdy]]$
§s %1 12 %0
$\supset \$HYPTH\, ([\lambda l.[\lambda e.GrpIdy]]\, l\, e)$
§\ $[\lambda l_{ggg}.[\lambda e_g.GrpIdy_o]_{(og)}] l_{ggg}$
$= ([\lambda l.[\lambda e.GrpIdy]]\, l)\, [\lambda e.GrpIdy]$
§s %1 6 %0
$\supset \$HYPTH\, ([\lambda e.GrpIdy]\, e)$
§\ $[\lambda e_g.GrpIdy_o] e_g$
$= ([\lambda e.GrpIdy]\, e)\, GrpIdy$
§s %1 3 %0
$\supset \$HYPTH\, GrpIdy$

use Proof Template A5215H (∀ I): H ⊃ ∀ x: B → H ⊃ B [x/a]
:= $\$T5215H$ g_τ
wff 1371 : g_τ := $\$T5215H$
:= $\$X5215H$ $a_{\$T5215H}$

wff 1375 : $a_{\$T5215H}$:= $\$X5215H$
:= $\$A5215H$ $f_{\$T5215H}$
wff 1444 : $f_{\$T5215H}$:= $\$A5215H$
:= $\$H5215H$ %0
wff 2082 : $\supset \$HYPTH\, GrpIdy_o$:= $\$H5215H$
<< A5215H.r0t.txt
:= $\$T5215H$
:= $\$X5215H$
:= $\$A5215H$
:= $\$H5215H$
%0
$\supset \$HYPTH\, (\wedge\, (=(l\, f\, e)\, f)\, (=(l\, e\, f)\, f))$
$\supset_{ooo}\$HYPTH_o(\wedge_{ooo}(=_{ogg}(l_{ggg}f_ge_g)f_g)(=_{ogg}(l_{ggg}e_gf_g)f_g))$

use Proof Template K8019H: H ⊃ (A ∧ B) → H ⊃ A, H ⊃ B
:= $\$H8019H$ %0
wff 2140 : $\supset \$HYPTH\, (\wedge\, (=(l\, f\, e)\, f)\, (=(l\, e\, f)\, f))_o$:= $\$H8019H$
<< K8019H.r0t.txt
:= $\$H8019H$
%$\$B8019H$
$\supset \$HYPTH\, (=(l\, e\, f)\, f)$:= $\$B8019H$
$\supset_{ooo}\$HYPTH_o(=_{ogg}(l_{ggg}e_gf_g)f_g)$:= $\$B8019H$
:= $\$A8019H$
:= $\$B8019H$

:= $\$FIDTY$ %0
wff 2206 : $\supset \$HYPTH\, (=(l\, e\, f)\, f)_o$:= $\$FIDTY$

.4: Proof of H ⊃ e = f

%$\$FIDTY$
$\supset \$HYPTH\, (=(l\, e\, f)\, f)$:= $\$FIDTY$
$\supset_{ooo}\$HYPTH_o(=_{ogg}(l_{ggg}e_gf_g)f_g)$:= $\$FIDTY$
:= $\$FIDTY$
%$\$EIDTY$
$\supset \$HYPTH\, (=(l\, e\, f)\, e)$:= $\$EIDTY$
$\supset_{ooo}\$HYPTH_o(=_{ogg}(l_{ggg}e_gf_g)e_g)$:= $\$EIDTY$
:= $\$EIDTY$
§s' %1 5 %0
$\supset \$HYPTH\, (=e\, f)$

use Proof Template K8025 (Deduction Theorem): (H ∧ I) ⊃ A → H ⊃ (I ⊃ A)
<< K8025.r0t.txt
%0
$\supset (\wedge\, (Grp\, g\, l)\, (GrpIdO\, g\, l\, e))\, (\supset (GrpIdO\, g\, l\, f)\, (=e\, f))$
$\supset_{ooo}(\wedge_{ooo}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}g_\tau l_{ggg})(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}g_\tau l_{ggg}e_g))\ldots$
$\ldots(\supset_{ooo}(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}g_\tau l_{ggg}f_g)(=_{ogg}e_gf_g))$

use Proof Template K8025 (Deduction Theorem): (H ∧ I) ⊃ A → H ⊃ (I ⊃ A)

<< K8025.r0t.txt
%0
$\supset (Grp\, g\, l)\, (\supset (GrpIdO\, g\, l\, e)\, (\supset (GrpIdO\, g\, l\, f)\, (= e\, f)))$
$\supset_{ooo}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau} g_\tau l_{ggg}) \ldots$
$\ldots (\supset_{ooo}(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau} g_\tau l_{ggg} e_g)(\supset_{ooo}(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau} g_\tau l_{ggg} f_g)(=_{ogg} e_g f_g)))$

:= $GrpIdElUniq$ %0
wff 4849 : $\supset (Grp\, g\, l)\, (\supset (GrpIdO\, g\, l\, e)\, (\supset (GrpIdO\, g\, l\, f)\, (= e\, f)))_{o,\ldots}$:= $GrpIdElUniq$

undefine local variables
:= $\$HYPTH$
:= $\$TMPDED$

##
Print Result
##

%0
$\supset (Grp\, g\, l)\, (\supset (GrpIdO\, g\, l\, e)\, (\supset (GrpIdO\, g\, l\, f)\, (= e\, f)))$:= $GrpIdElUniq$
$\supset_{ooo}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau} g_\tau l_{ggg}) \ldots$
$\ldots (\supset_{ooo}(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau} g_\tau l_{ggg} e_g)(\supset_{ooo}(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau} g_\tau l_{ggg} f_g)(=_{ogg} e_g f_g)))$:= $GrpIdElUniq$

5.1.94 Results for File natural_numbers.r0.txt

##
Peano's Postulates
##
##
Source: [Andrews 2002 (ISBN 1-4020-0763-9), p. 258 f.]
##

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##
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##

<< basics.r0.txt

variables used
t: domain (type of the natural numbers)
z: zero
s: successor function
n: set of natural numbers

:= $\$F5222$

:= $XorCaseTRight$ %0
wff 1721 : $=(XOR\,x\,T)\,(\sim x)_{o,\ldots}$:= $XorCaseTRight$

.c: (T X A) = (A X T)

%$XorCaseTRight$
$=(XOR\,x\,T)\,(\sim x)$:= $XorCaseTRight$
$=_{ooo}(XOR_{ooo}x_oT_o)(\sim_{oo}x_o)$:= $XorCaseTRight$

use Proof Template A5201b (Swap): A = B → B = A
<< A5201b.r0t.txt
%0
$=(\sim x)\,(XOR\,x\,T)$
$=_{ooo}(\sim_{oo}x_o)(XOR_{ooo}x_oT_o)$

%$XorCaseTLeft$
$=(XOR\,T\,x)\,(\sim x)$:= $XorCaseTLeft$
$=_{ooo}(XOR_{ooo}T_ox_o)(\sim_{oo}x_o)$:= $XorCaseTLeft$
§s %0 3 %1
$=(XOR\,T\,x)\,(XOR\,x\,T)$

:= $XorCaseTLeftRight$ %0
wff 1799 : $=(XOR\,T\,x)\,(XOR\,x\,T)_o$:= $XorCaseTLeftRight$

5.1.112 Results for File xor_group.r0.txt

##
Group Property of Exclusive Disjunction (Exclusive OR, XOR)
##
##
Source: [Kubota 2015 (doi: 10.4444/100.10)]
##
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Written by Ken Kubota (<mail@kenkubota.de>).
##
This file is part of the work “On the Theory of Mathematical Forms”.
For more information visit: <http://dx.doi.org/10.4444/100.10>
##

<< A5229.r0.txt
<< group.r0.txt
<< xor_associativity.r0.txt
<< xor_identity_element.r0.txt
<< xor_inverse_element.r0.txt

shorthands

:= $\$Xab\ \ XOR_{ooo}a_ob_o$
\# wff 1707 : $XOR\,a\,b_o$:= $\$Xab$
:= $\$Xbc\ \ XOR_{ooo}b_oc_o$
\# wff 1712 : $XOR\,b\,c_o$:= $\$Xbc$
:= $\$GrpAss\ \ \forall_{o(o\backslash 3)\tau}o_\tau \ldots$
$\ldots[\lambda a_o.(\forall_{o(o\backslash 3)\tau}o_\tau[\lambda b_o.(\forall_{o(o\backslash 3)\tau}o_\tau[\lambda c_o.(=_{ooo}(l_{ooo}(l_{ooo}a_ob_o)c_o)(l_{ooo}a_o(l_{ooo}b_oc_o)))_o])_o])_o]$
\# wff 6925 : $\forall\,o\,[\lambda a.(\forall\,o\,[\lambda b.(\forall\,o\,[\lambda c.(=(l\,(l\,a\,b)\,c)\,(l\,a\,(l\,b\,c)))])])]_o$:= $\$GrpAss$
:= $\$GrpIdy\ \ \forall_{o(o\backslash 3)\tau}o_\tau[\lambda a_o.(\wedge_{ooo}(=_{ooo}(l_{ooo}a_oe_o)a_o)(=_{ooo}(l_{ooo}e_oa_o)a_o))_o]$
\# wff 6936 : $\forall\,o\,[\lambda a.(\wedge\,(=(l\,a\,e)\,a)\,(=(l\,e\,a)\,a))]_o$:= $\$GrpIdy$
:= $\$GrpInv\ \ \forall_{o(o\backslash 3)\tau}o_\tau[\lambda a_o.(\exists_{o(o\backslash 3)\tau}o_\tau[\lambda b_o.(\wedge_{ooo}(=_{ooo}(l_{ooo}a_ob_o)e_o)(=_{ooo}(l_{ooo}b_oa_o)e_o))_o])_o]$
\# wff 6939 : $\forall\,o\,[\lambda a.(\exists\,o\,[\lambda b.(\wedge\,(=(l\,a\,b)\,e)\,(=(l\,b\,a)\,e))])]_o$:= $\$GrpInv$
:= $\$XAss\ \ \forall_{o(o\backslash 3)\tau}o_\tau \ldots$
$\ldots[\lambda a_o.(\forall_{o(o\backslash 3)\tau}o_\tau[\lambda b_o.(\forall_{o(o\backslash 3)\tau}o_\tau[\lambda c_o.(=_{ooo}(XOR_{ooo}\$Xab_oc_o)(XOR_{ooo}a_o\$Xbc_o))_o])_o])_o]$
\# wff 2616 : $\forall\,o\,[\lambda a.(\forall\,o\,[\lambda b.(\forall\,o\,[\lambda c.(=(XOR\,\$Xab\,c)\,(XOR\,a\,\$Xbc))])])]_{o,\ldots}$:= $\$XAss\ \ XorAssociativity$
:= $\$XIdy\ \ \forall_{o(o\backslash 3)\tau}o_\tau[\lambda a_o.(\wedge_{ooo}(=_{ooo}(XOR_{ooo}a_oe_o)a_o)(=_{ooo}(XOR_{ooo}e_oa_o)a_o))_o]$
\# wff 6950 : $\forall\,o\,[\lambda a.(\wedge\,(=(XOR\,a\,e)\,a)\,(=(XOR\,e\,a)\,a))]_o$:= $\$XIdy$
:= $\$XInv\ \ \forall_{o(o\backslash 3)\tau}o_\tau[\lambda a_o.(\exists_{o(o\backslash 3)\tau}o_\tau[\lambda b_o.(\wedge_{ooo}(=_{ooo}\$Xab_oe_o)(=_{ooo}(XOR_{ooo}b_oa_o)e_o))_o])_o]$
\# wff 6953 : $\forall\,o\,[\lambda a.(\exists\,o\,[\lambda b.(\wedge\,(=\$Xab\,e)\,(=(XOR\,b\,a)\,e))])]_o$:= $\$XInv$
:= $\$XFIdy\ \ \forall_{o(o\backslash 3)\tau}o_\tau[\lambda a_o.(\wedge_{ooo}(=_{ooo}(XOR_{ooo}a_oF_o)a_o)(=_{ooo}(XOR_{ooo}F_oa_o)a_o))_o]$
\# wff 2850 : $\forall\,o\,[\lambda a.(\wedge\,(=(XOR\,a\,F)\,a)\,(=(XOR\,F\,a)\,a))]_o$:= $\$XFIdy\ \ XorIdentityElement$
:= $\$XFInv\ \ \forall_{o(o\backslash 3)\tau}o_\tau[\lambda a_o.(\exists_{o(o\backslash 3)\tau}o_\tau[\lambda b_o.(\wedge_{ooo}(=_{ooo}\$Xab_oF_o)(=_{ooo}(XOR_{ooo}b_oa_o)F_o))_o])_o]$
\# wff 6905 : $\forall\,o\,[\lambda a.(\exists\,o\,[\lambda b.(\wedge\,(=\$Xab\,F)\,(=(XOR\,b\,a)\,F))])]_{o,\ldots}$:= $\$XFInv\ \ XorInverseElement$

\#\# .1

§= $Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_\tau XOR_{ooo}$
\# $=(Grp\,o\,XOR)\,(Grp\,o\,XOR)$
§\ $Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_\tau$
\# $=(Grp\,o)\,[\lambda l.(\wedge\,\$GrpAss\,(\exists\,o\,[\lambda e.(\wedge\,\$GrpIdy\,\$GrpInv)]))]$
§s %1 6 %0
\# $=(Grp\,o\,XOR)\,([\lambda l.(\wedge\,\$GrpAss\,(\exists\,o\,[\lambda e.(\wedge\,\$GrpIdy\,\$GrpInv)]))]\,XOR)$
§\ $[\lambda l_{ooo}.(\wedge_{ooo}\$GrpAss_o(\exists_{o(o\backslash 3)\tau}o_\tau[\lambda e_o.(\wedge_{ooo}\$GrpIdy_o\$GrpInv_o)_o]))_o]XOR_{ooo}$
\# $=([\lambda l.(\wedge\,\$GrpAss\,(\exists\,o\,[\lambda e.(\wedge\,\$GrpIdy\,\$GrpInv)]))]\,XOR)\ \ldots$
$\ldots(\wedge\,\$XAss\,(\exists\,o\,[\lambda e.(\wedge\,\$XIdy\,\$XInv)]))$
§s %1 3 %0
\# $=(Grp\,o\,XOR)\,(\wedge\,\$XAss\,(\exists\,o\,[\lambda e.(\wedge\,\$XIdy\,\$XInv)]))$

:= $\$T1$ %0
\# wff 6977 : $=(Grp\,o\,XOR)\,(\wedge\,\$XAss\,(\exists\,o\,[\lambda e.(\wedge\,\$XIdy\,\$XInv)]))_o$:= $\$T1$

\#\# .2

§= $[\lambda e_o.(\wedge_{ooo}\$XIdy_o\$XInv_o)_o]F_o$
\# $=([\lambda e.(\wedge\,\$XIdy\,\$XInv)]\,F)\,([\lambda e.(\wedge\,\$XIdy\,\$XInv)]\,F)$
§\ $[\lambda e_o.(\wedge_{ooo}\$XIdy_o\$XInv_o)_o]F_o$
\# $=([\lambda e.(\wedge\,\$XIdy\,\$XInv)]\,F)\,(\wedge\,\$XFIdy\,\$XFInv)$

§s %1 3 %0
\# $=([\lambda e.(\wedge \$XIdy\,\$XInv)]\,F)\,(\wedge \$XFIdy\,\$XFInv)$

:= $\$T2$ %0
\# wff 6983 : $=([\lambda e.(\wedge \$XIdy\,\$XInv)]\,F)\,(\wedge \$XFIdy\,\$XFInv)_o$:= $\$T2$

\#\# .3

%$\$XFIdy$
\# $\forall o\,[\lambda a.(\wedge\,(=(XOR\,a\,F)\,a)\,(=(XOR\,F\,a)\,a))]$:= $\$XFIdy$ $XorIdentityElement$
\# $\forall_{o(o\backslash 3)\tau} o_\tau[\lambda a_o.(\wedge_{ooo}(=_{ooo}(XOR_{ooo}a_oF_o)a_o)(=_{ooo}(XOR_{ooo}F_oa_o)a_o))_o]$:= $\$XFIdy$ $XorIdentityElement$
\#\# use Proof Template A5219b (Rule T): A → A = T
:= $\$A5219b$ %0
\# wff 2850 : $\forall o\,[\lambda a.(\wedge\,(=(XOR\,a\,F)\,a)\,(=(XOR\,F\,a)\,a))]_o$:= $\$A5219b$ $\$XFIdy$ $XorIdentityElement$
<< A5219b.r0t.txt
:= $\$A5219b$

:= $\$E$ %0
\# wff 7000 : $=\$XFIdy\,T_o$:= $\$E$

%$\$T2$
\# $=([\lambda e.(\wedge \$XIdy\,\$XInv)]\,F)\,(\wedge \$XFIdy\,\$XFInv)$:= $\$T2$
\# $=_{o\omega\omega}([\lambda e_o.(\wedge_{ooo}\$XIdy_o\$XInv_o)_o]F_o)(\wedge_{ooo}\$XFIdy_o\$XFInv_o)$:= $\$T2$
:= $\$T2$
%$\$E$
\# $=\$XFIdy\,T$:= $\$E$
\# $=_{ooo}\$XFIdy_oT_o$:= $\$E$
:= $\$E$
§s %1 13 %0
\# $=([\lambda e.(\wedge \$XIdy\,\$XInv)]\,F)\,(\wedge\,T\,\$XFInv)$

:= $\$T3$ %0
\# wff 7002 : $=([\lambda e.(\wedge \$XIdy\,\$XInv)]\,F)\,(\wedge\,T\,\$XFInv)_o$:= $\$T3$

\#\# .4

%$\$XFInv$
\# $\forall o\,[\lambda a.(\exists\,o\,[\lambda b.(\wedge\,(=\$Xab\,F)\,(=(XOR\,b\,a)\,F))])]$:= $\$XFInv$ $XorInverseElement$
\# $\forall_{o(o\backslash 3)\tau} o_\tau[\lambda a_o.(\exists_{o(o\backslash 3)\tau} o_\tau[\lambda b_o.(\wedge_{ooo}(=_{ooo}\$Xab_oF_o)(=_{ooo}(XOR_{ooo}b_oa_o)F_o))_o])_o]$
:= $\$XFInv$ $XorInverseElement$
\#\# use Proof Template A5219b (Rule T): A → A = T
:= $\$A5219b$ %0
\# wff 6905 : $\forall o\,[\lambda a.(\exists\,o\,[\lambda b.(\wedge\,(=\$Xab\,F)\,(=(XOR\,b\,a)\,F))])]_{o,\dots}$:= $\$A5219b$ $\$XFInv$ $XorInverseElement$
<< A5219b.r0t.txt

:= $\$A5219b$

:= $\$E$ %0
wff 7019 : $=\$XFInv\,T_o$:= $\$E$

%$\$T3$
$=([\lambda e.(\wedge\,\$XIdy\,\$XInv)]\,F)\,(\wedge\,T\,\$XFInv)$:= $\$T3$
$=_{o\omega\omega}([\lambda e_o.(\wedge_{ooo}\$XIdy_o\$XInv_o)_o]F_o)(\wedge_{ooo}T_o\$XFInv_o)$:= $\$T3$
:= $\$T3$
%$\$E$
$=\$XFInv\,T$:= $\$E$
$=_{ooo}\$XFInv_oT_o$:= $\$E$
:= $\$E$
§s %1 7 %0
$=([\lambda e.(\wedge\,\$XIdy\,\$XInv)]\,F)\,A5212$

.5

%$A5211$
$=A5212\,T$:= $A5211$ $A5229a$
$=_{ooo}A5212_oT_o$:= $A5211$ $A5229a$
§s %1 3 %0
$=([\lambda e.(\wedge\,\$XIdy\,\$XInv)]\,F)\,T$
use Proof Template A5201b (Swap): A = B → B = A
<< A5201b.r0t.txt
%0
$=T\,([\lambda e.(\wedge\,\$XIdy\,\$XInv)]\,F)$
$=_{o\omega\omega}T_\omega([\lambda e_o.(\wedge_{ooo}\$XIdy_o\$XInv_o)_o]F_o)$
%T
$===$:= $A5200t$ T
$=_{o\omega\omega}=_\omega=_\omega$:= $A5200t$ T
§s %0 1 %1
$[\lambda e.(\wedge\,\$XIdy\,\$XInv)]\,F$

.6

use Proof Template K8031 (∃ Gen): ([\x.B]A) → ∃ x: B
:= $\$T8031$ o
wff 2 : o_τ := $\$T8031$
:= $\$B8031$ %0/2
wff 6973 : $[\lambda e.(\wedge\,\$XIdy\,\$XInv)]_{oo}$:= $\$B8031$
:= $\$A8031$ %0/3
wff 20 : $=[\lambda x.T]\,[\lambda x.x]_{o,\ldots}$:= $\$A8031$ F
:= $\$P8031$ $\$B8031_{oo}F_o$
wff 6978 : $\$B8031\,F_{o,\ldots}$:= $\$P8031$
<< K8031.r0t.txt
:= $\$T8031$
:= $\$B8031$
:= $\$A8031$

:= $\$T6$ %0
wff 6974 : $\exists\, o\, [\lambda e.(\wedge\, \$XIdy\, \$XInv)]_{o,\ldots}$:= $\$T6$

.7

%$\$T1$
$= (Grp\, o\, XOR)\, (\wedge\, \$XAss\, \$T6)$:= $\$T1$
$=_{o\omega\omega}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau} o_\tau XOR_{ooo})(\wedge_{ooo}\$XAss_o\$T6_o)$:= $\$T1$
%$\$T6$
$\exists\, o\, [\lambda e.(\wedge\, \$XIdy\, \$XInv)]$:= $\$T6$
$\exists_{o(o\backslash 3)\tau} o_\tau[\lambda e_o.(\wedge_{ooo}\$XIdy_o\$XInv_o)_o]$:= $\$T6$
:= $\$T6$

use Proof Template A5219b (Rule T): A → A = T
:= $\$A5219b$ %0
wff 6974 : $\exists\, o\, [\lambda e.(\wedge\, \$XIdy\, \$XInv)]_{o,\ldots}$:= $\$A5219b$
<< A5219b.r0t.txt
:= $\$A5219b$

:= $\$TMP$ %0
wff 7654 : $= (\exists\, o\, [\lambda e.(\wedge\, \$XIdy\, \$XInv)])\, T_o$:= $\$TMP$

%$\$T1$
$= (Grp\, o\, XOR)\, (\wedge\, \$XAss\, (\exists\, o\, [\lambda e.(\wedge\, \$XIdy\, \$XInv)]))$:= $\$T1$
$=_{o\omega\omega}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau} o_\tau XOR_{ooo})(\wedge_{ooo}\$XAss_o(\exists_{o(o\backslash 3)\tau} o_\tau[\lambda e_o.(\wedge_{ooo}\$XIdy_o\$XInv_o)_o]))$
:= $\$T1$
:= $\$T1$
%$\$TMP$
$= (\exists\, o\, [\lambda e.(\wedge\, \$XIdy\, \$XInv)])\, T$:= $\$TMP$
$=_{ooo}(\exists_{o(o\backslash 3)\tau} o_\tau[\lambda e_o.(\wedge_{ooo}\$XIdy_o\$XInv_o)_o])T_o$:= $\$TMP$
:= $\$TMP$
§s %1 7 %0
$= (Grp\, o\, XOR)\, (\wedge\, \$XAss\, T)$

:= $\$TMP$ %0
wff 7656 : $= (Grp\, o\, XOR)\, (\wedge\, \$XAss\, T)_o$:= $\$TMP$

%$\$XAss$
$\forall\, o\, [\lambda a.(\forall\, o\, [\lambda b.(\forall\, o\, [\lambda c.(= (XOR\, \$Xab\, c)\, (XOR\, a\, \$Xbc))])])]$:= $\$XAss$
$XorAssociativity$
$\forall_{o(o\backslash 3)\tau} o_\tau \ldots$
$\ldots [\lambda a_o.(\forall_{o(o\backslash 3)\tau} o_\tau[\lambda b_o.(\forall_{o(o\backslash 3)\tau} o_\tau[\lambda c_o.(=_{ooo}(XOR_{ooo}\$Xab_oc_o)(XOR_{ooo}a_o\$Xbc_o))_o])_o])_o]$:=
$\$XAss$ $XorAssociativity$

use Proof Template A5219b (Rule T): A → A = T
:= $\$A5219b$ %0
wff 2616 : $\forall\, o\, [\lambda a.(\forall\, o\, [\lambda b.(\forall\, o\, [\lambda c.(= (XOR\, \$Xab\, c)\, (XOR\, a\, \$Xbc))])])]_{o,\ldots}$:=
$\$A5219b$ $\$XAss$ $XorAssociativity$

<< A5219b.r0t.txt
:= $\$A5219b$

%$\$TMP$
$=(Grp\,o\,XOR)\,(\wedge\,\$XAss\,T)$:= $\$TMP$
$=_{o\omega\omega}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau}XOR_{ooo})(\wedge_{ooo}\$XAss_oT_o)$:= $\$TMP$
:= $\$TMP$
%1
$=\$XAss\,T$
$=_{ooo}\$XAss_oT_o$
§s %1 13 %0
$=(Grp\,o\,XOR)\,A5212$

%$A5211$
$=A5212\,T$:= $A5211$ $A5229a$
$=_{ooo}A5212_oT_o$:= $A5211$ $A5229a$
§s %1 3 %0
$=(Grp\,o\,XOR)\,T$
use Proof Template A5201b (Swap): A = B → B = A
<< A5201b.r0t.txt
%0
$=T\,(Grp\,o\,XOR)$
$=_{o\omega\omega}T_{\omega}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau}XOR_{ooo})$
%T
$===$:= $A5200t$ T
$=_{o\omega\omega}=_{\omega}=_{\omega}$:= $A5200t$ T
§s %0 1 %1
$Grp\,o\,XOR$

:= $XorGroup$ %0
wff 6955 : $Grp\,o\,XOR_{o,\dots}$:= $XorGroup$

demonstrate that XOR now has type Grp_o
§= $_{Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau}}$ XOR
$=XOR\,XOR$
%0
$=XOR\,XOR$
$=_{o\,(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau})\,(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau})}XOR_{Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau}}XOR_{Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau}}$

demonstrate that Grp_o now has type tau (type “type”)
§= $_{\tau}$ $Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau}$
$=(Grp\,o)\,(Grp\,o)$
%0
$=(Grp\,o)\,(Grp\,o)$
$=_{o\tau\tau}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau})(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau})$

undefine local variables
:= $\$Xab$
:= $\$Xbc$

:= $\$GrpAss$
:= $\$GrpIdy$
:= $\$GrpInv$
:= $\$XAss$
:= $\$XIdy$
:= $\$XInv$
:= $\$XFIdy$
:= $\$XFInv$

5.1.113 Results for File xor_group_identity_element_unique.r0.txt

##
Uniqueness of the Group Identity Element of the XOR Group
##
##
Source: [Kubota 2015 (doi: 10.4444/100.10)]
##

Written by Ken Kubota (<mail@kenkubota.de>).
##
This file is part of the work "On the Theory of Mathematical Forms".
For more information visit: <http://dx.doi.org/10.4444/100.10>
##

<< A5223.r0.txt
<< group_identity_element_unique.r0.txt
<< xor_group.r0.txt

shorthands
:= $\$GIdOXe\ \ GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau}XOR_{ooo}e_o$
wff 8464 : $GrpIdO\,o\,XOR\,e_o$:= $\$GIdOXe$
:= $\$GIdOXf\ \ GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}o_{\tau}XOR_{ooo}f_o$
wff 8466 : $GrpIdO\,o\,XOR\,f_o$:= $\$GIdOXf$

.1

$\%GrpIdElUniq$
$\supset (Grp\,g\,l)\,(\supset (GrpIdO\,g\,l\,e)\,(\supset (GrpIdO\,g\,l\,f)\,(= e\,f)))$:= $GrpIdElUniq$
$\supset_{ooo}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}g_{\tau}l_{ggg})\ldots$
$\ldots(\supset_{ooo}(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}g_{\tau}l_{ggg}e_g)(\supset_{ooo}(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}g_{\tau}l_{ggg}f_g)(=_{ogg}e_gf_g)))$:=
$GrpIdElUniq$

use Proof Template A5221 (Sub): B → B [x/A]
:= $\$B5221$ %0
wff 4849 : $\supset (Grp\,g\,l)\,(\supset (GrpIdO\,g\,l\,e)\,(\supset (GrpIdO\,g\,l\,f)\,(= e\,f)))_{o,\ldots}$:= $\$B5221$
$GrpIdElUniq$
:= $\$T5221\ \ \tau$
wff 0 : τ_{τ} := $\$T5221$

:= $\$X5221\ g_\tau$
wff 1411 : g_τ := $\$X5221$
:= $\$A5221\ o$
wff 2 : o_τ := $\$A5221$
<< A5221.r0t.txt
:= $\$B5221$
:= $\$T5221$
:= $\$X5221$
:= $\$A5221$
%0
$\supset (Grp\,o\,l)\,(\supset (GrpIdO\,o\,l\,e)\,(\supset (GrpIdO\,o\,l\,f)\,(=e\,f)))$
$\supset_{ooo}(Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_\tau l_{ooo})\ldots$
$\ldots(\supset_{ooo}(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}o_\tau l_{ooo}e_o)(\supset_{ooo}(GrpIdO_{o\backslash 3(\backslash 4\backslash 4\backslash 3)\tau}o_\tau l_{ooo}f_o)(=_{ooo}e_of_o)))$

use Proof Template A5221 (Sub): B → B [x/A]
:= $\$B5221$ %0
wff 8515 : $\supset (Grp\,o\,l)\,(\supset (GrpIdO\,o\,l\,e)\,(\supset (GrpIdO\,o\,l\,f)\,(=e\,f)))_{o,\ldots}$:= $\$B5221$
:= $\$T5221\ ooo$
wff 35 : ooo_τ := $\$T5221$
:= $\$X5221\ l_{\$T5221}$
wff 6055 : $l_{\$T5221}$:= $\$X5221$
:= $\$A5221\ [\lambda x_o.[\lambda y_o.(\sim_{oo}(=_{\$T5221}x_oy_o))_o]_{(oo)}]$
wff 135 : $[\lambda x.[\lambda y.(\sim(=x\,y))]]_{\$T5221,\ldots}$:= $\$A5221\quad XOR$
<< A5221.r0t.txt
:= $\$B5221$
:= $\$T5221$
:= $\$X5221$
:= $\$A5221$

:= $\$TMP$ %0
wff 8564 : $\supset XorGroup\,(\supset \$GIdOXe\,(\supset \$GIdOXf\,(=e\,f)))_{o,\ldots}$:= $\$TMP$

.2

%$XorGroup$
$Grp\,o\,XOR$:= $XorGroup$
$Grp_{o(\backslash 4\backslash 4\backslash 3)\tau}o_\tau XOR_{ooo}$:= $XorGroup$

use Proof Template A5219b (Rule T): A → A = T
:= $\$A5219b$ %0
wff 7733 : $Grp\,o\,XOR_{o,\ldots}$:= $\$A5219b\quad XorGroup$
<< A5219b.r0t.txt
:= $\$A5219b$
%0
$=XorGroup\,T$
$=_{ooo}XorGroup_oT_o$

%$\$TMP$
$\supset XorGroup\,(\supset \$GIdOXe\,(\supset \$GIdOXf\,(=e\,f)))$:= $\$TMP$

\# $\supset_{ooo} XorGroup_o(\supset_{ooo}\$GIdOXe_o(\supset_{ooo}\$GIdOXf_o(=_{ooo}e_of_o)))$:= $\$TMP$
:= $\$TMP$
%1
\# $= XorGroup\,T$
\# $=_{ooo} XorGroup_o T_o$
§s %1 5 %0
\# $\supset T\,(\supset \$GIdOXe\,(\supset \$GIdOXf\,(= e\,f)))$

:= $\$TMP$ %0
\# wff 8584 : $\supset T\,(\supset \$GIdOXe\,(\supset \$GIdOXf\,(= e\,f)))_o$:= $\$TMP$

\#\# use Proof Template A5221 (Sub): B → B [x/A]
:= $\$B5221$ $=_{ooo}(\supset_{ooo} T_o y_o) y_o$
\# wff 823 : $= (\supset T\,y)\,y_{o,\ldots}$:= $\$B5221$ $A5223$
:= $\$T5221$ o
\# wff 2 : o_τ := $\$T5221$
:= $\$X5221$ y_o
\# wff 34 : y_o := $\$X5221$
:= $\$A5221$ %0/3
\# wff 8563 : $\supset \$GIdOXe\,(\supset \$GIdOXf\,(= e\,f))_o$:= $\$A5221$
<< A5221.r0t.txt
:= $\$B5221$
:= $\$T5221$
:= $\$X5221$
:= $\$A5221$
%0
\# $= \$TMP\,(\supset \$GIdOXe\,(\supset \$GIdOXf\,(= e\,f)))$
\# $=_{ooo}\$TMP_o(\supset_{ooo}\$GIdOXe_o(\supset_{ooo}\$GIdOXf_o(=_{ooo}e_of_o)))$

%$\$TMP$
\# $\supset T\,(\supset \$GIdOXe\,(\supset \$GIdOXf\,(= e\,f)))$:= $\$TMP$
\# $\supset_{ooo} T_o(\supset_{ooo}\$GIdOXe_o(\supset_{ooo}\$GIdOXf_o(=_{ooo}e_of_o)))$:= $\$TMP$
:= $\$TMP$
%1
\# $= (\supset T\,(\supset \$GIdOXe\,(\supset \$GIdOXf\,(= e\,f))))\,(\supset \$GIdOXe\,(\supset \$GIdOXf\,(= e\,f)))$
\# $=_{ooo}(\supset_{ooo} T_o(\supset_{ooo}\$GIdOXe_o(\supset_{ooo}\$GIdOXf_o(=_{ooo}e_of_o))))\ldots$
$\ldots(\supset_{ooo}\$GIdOXe_o(\supset_{ooo}\$GIdOXf_o(=_{ooo}e_of_o)))$
§s %1 1 %0
\# $\supset \$GIdOXe\,(\supset \$GIdOXf\,(= e\,f))$

\#\# use Proof Template K8026 (Deduction Theorem Reversed): H ⊃ (I ⊃ A) → (H ∧ I) ⊃ A
<< K8026.r0t.txt
%0
\# $\supset (\wedge \$GIdOXe\,\$GIdOXf)\,(= e\,f)$
\# $\supset_{ooo}(\wedge_{ooo}\$GIdOXe_o\$GIdOXf_o)(=_{ooo}e_of_o)$

:= $XorGrpIdElUniq$ %0
\# wff 8693 : $\supset (\wedge \$GIdOXe\,\$GIdOXf)\,(= e\,f)_{o,\ldots}$:= $XorGrpIdElUniq$

undefine local variables
:= $\$GIdOXe$
:= $\$GIdOXf$

5.1.114 Results for File xor_identity_element.r0.txt

##
Neutral Element of Exclusive Disjunction (Exclusive OR, XOR)
##
##
Source: [Kubota 2015 (doi: 10.4444/100.10)]
##

Written by Ken Kubota (<mail@kenkubota.de>).
##
This file is part of the work "On the Theory of Mathematical Forms".
For more information visit: <http://dx.doi.org/10.4444/100.10>
##

<< basics.r0.txt
<< xor_case_f.r0.txt

$\%XorCaseFRight$
\# $= (XOR\,x\,F)\,x$:= $XorCaseFRight$
\# $=_{ooo}(XOR_{ooo}x_oF_o)x_o$:= $XorCaseFRight$
$\%XorCaseFLeft$
\# $= (XOR\,F\,x)\,x$:= $XorCaseFLeft$
\# $=_{ooo}(XOR_{ooo}F_ox_o)x_o$:= $XorCaseFLeft$

use Proof Template K8020: A, B → A ∧ B
:= $\$A8020$ %1
\# wff 1718 : $= (XOR\,x\,F)\,x_{o,\ldots}$:= $\$A8020$ $XorCaseFRight$
:= $\$B8020$ %0
\# wff 1642 : $= (XOR\,F\,x)\,x_{o,\ldots}$:= $\$B8020$ $XorCaseFLeft$
<< K8020.r0t.txt
:= $\$A8020$
:= $\$B8020$
%0
\# $\wedge\,XorCaseFRight\,XorCaseFLeft$
\# $\wedge_{ooo}XorCaseFRight_oXorCaseFLeft_o$

use Proof Template A5220 (Gen): A → ∀ x: A
:= $\$T5220$ o
\# wff 2 : o_τ := $\$T5220$
:= $\$X5220$ x_o
\# wff 16 : x_o := $\$X5220$
:= $\$A5220$ %0

```
<< A5201b.r0t.txt
%0

§s %4 3 %0

:= $GF; := $HG; := $HxGF; := $HGxF


##
##  Print Result
##

%0
```

5.2.140 File definitions1.r0.txt

```
##
##  Basic Definitions
##
##
##  Source: [Andrews 2002 (ISBN 1-4020-0763-9), p. 212]
##
##  Copyright (c) 2015 Owl of Minerva Press GmbH. All rights reserved.
##  Written by Ken Kubota (<mail@kenkubota.de>).
##
##  This file is part of the work "On the Theory of Mathematical Forms".
##  For more information visit: <http://dx.doi.org/10.4444/100.10>
##

## Definition of truth
:= T ((={{{o,@},@}}_={@}){{o,@}}_={@})

## Definition of falsehood
:= F ((={{o,{o,o}},{o,o}}_[\x{o}{o}.T{o}]{o,o}){{o,{o,o}}}_[\x{o}{o}.x{o}{o}]{o,o})

## Definition of the universal quantifier (with type abstraction)
:= A [\t{^}{^}.[\p{{o,t{^}}}{{o,t{^}}}.((={{{o,{o,t{^}}},{o,t{^}}}}_[\x{t{^}}{t{^}}.
T{o}]{{o,t{^}}}){{o,{o,t{^}}}}_p{{o,t{^}}}{{o,t{^}}}){o}]{{o,{o,t{^}}}}]

## Definition of the conjunction
:= & [\x{o}{o}.[\y{o}{o}.((={{{o,@},@}}_[\g{{{o,o},o}}{{{o,o},o}}.((g{{{o,o},o}}{{{o
,o},o}}_T{o}){{o,o}}_T{o}){o}]{@}){{o,@}}_[\g{{{o,o},o}}{{{o,o},o}}.((g{{{o,o},o}}{{
{o,o},o}}_x{o}{o}){{o,o}}_y{o}{o}){o}]{@}){o}]{{o,o}}]

## Definition of the implication
:= => [\x{o}{o}.[\y{o}{o}.((={{{o,o},o}}_x{o}{o}){{o,o}}_((&{{{o,o},o}}_x{o}{o}){{o,
o}}_y{o}{o}){o}){o}]{{o,o}}]

## Definition of the negation
```

```
:= ! [\a{o}{o}.((={{{o,o},o}}_F{o}){{o,o}}_a{o}{o}){o}]

## Definition of the disjunction
:= | [\a{o}{o}.[\b{o}{o}.(!{{o,o}}_((&{{{o,o},o}}_(!{{o,o}}_a{o}{o}){o}){{o,o}}_(!{{
o,o}}_b{o}{o}){o}){o}){o}]{{o,o}}]

## Definition of the existential quantifier (with type abstraction)
:= E [\t{^}{^}.[\p{{o,t{^}}}{{o,t{^}}}.(!{{o,o}}_((={{{o,{o,t{^}}},{o,t{^}}}}_[\x{t{
^}}{t{^}}.T{o}]{{o,t{^}}}){{o,{o,t{^}}}}_[\x{t{^}}{t{^}}.(!{{o,o}}_(p{{o,t{^}}}{{o,t
{^}}}_x{t{^}}{t{^}}){o}){o}]{{o,t{^}}}){o}){o}]{{o,{o,t{^}}}}]

## Definition of inequality
:= != [\x{@}{@}.[\y{@}{@}.(!{{o,o}}_((={{{o,@},@}}_x{@}{@}){{o,@}}_y{@}{@}){o}){o}]{
{o,@}}]
```

5.2.141 File definitions2.r0.txt

```
##
##  Further Definitions
##
##
##  Source: [Andrews 2002 (ISBN 1-4020-0763-9), pp. 231, 233]
##
##  Copyright (c) 2015 Owl of Minerva Press GmbH. All rights reserved.
##  Written by Ken Kubota (<mail@kenkubota.de>).
##
##  This file is part of the work "On the Theory of Mathematical Forms".
##  For more information visit: <http://dx.doi.org/10.4444/100.10>
##

<< definitions1.r0.txt

## Definition of the subset
:= SBSET [\t{^}{^}.[\x{{o,t{^}}}{{o,t{^}}}.[\y{{o,t{^}}}{{o,t{^}}}.((A{{{o,{o,\3{^}}
},^}}_t{^}{^}){{o,{o,t{^}}}}_[\z{t{^}}{t{^}}.((=>{{{o,o},o}}_(x{{o,t{^}}}{{o,t{^}}}_
z{t{^}}{t{^}}){o}){{o,o}}_(y{{o,t{^}}}{{o,t{^}}}_z{t{^}}{t{^}}){o}){o}]{{o,t{^}}}){o
}]{{o,{o,t{^}}}}]{{{o,{o,t{^}}},{o,t{^}}}}]

## Definition of the power set
:= PWSET [\t{^}{^}.[\y{{o,t{^}}}{{o,t{^}}}.[\x{{o,t{^}}}{{o,t{^}}}.(((SBSET{{{{o,{o,
\4{^}}},{o,\3{^}}},^}}_t{^}{^}){{{o,{o,t{^}}},{o,t{^}}}}_x{{o,t{^}}}{{o,t{^}}}){{o,{
o,t{^}}}}_y{{o,t{^}}}{{o,t{^}}}){o}]{{o,{o,t{^}}}}]{{{o,{o,t{^}}},{o,t{^}}}}]

## Definition of the uniqueness quantifier (with type abstraction)
:= E1 [\t{^}{^}.[\p{{o,t{^}}}{{o,t{^}}}.((E{{{o,{o,\3{^}}},^}}_t{^}{^}){{o,{o,t{^}}}
}_[\y{t{^}}{t{^}}.((={{{o,{o,t{^}}},{o,t{^}}}}_p{{o,t{^}}}{{o,t{^}}}){{o,{o,t{^}}}}_
(={{{o,t{^}},t{^}}}_y{t{^}}{t{^}}){o,t{^}}){o}]{{o,t{^}}}){o}]{{o,{o,t{^}}}}]
```