On the Theory of Mathematical Forms

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2015

:= $A5311$ %0
wff 5461 : $\supset (\exists_1 t\,[\lambda y.(p\,y)])\,(p\,(\iota\,p))_o$:= $A5311$

##
Q.E.D.
##

%0
$\supset (\exists_1 t\,[\lambda y.(p\,y)])\,(p\,(\iota\,p))$:= $A5311$
$\supset_{ooo}(\exists_{1o(o\backslash 3)\tau} t_\tau[\lambda y_t.(p_{ot}y_t)_o])(p_{ot}(\iota_{t(ot)}p_{ot}))$:= $A5311$

5.1.44 Results for File A5313.r0.txt

##
Proof A5313: (C_t_x_y_T = x) ∧ (C_t_x_y_F = y)
##
##
Source: [Andrews 2002 (ISBN 1-4020-0763-9), p. 235 f.]
##

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##
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##

##
"C[t]xyp can be read 'if p then x, else y'." [Andrews 2002, p. 235]
##

<< basics.r0.txt
<< A5205.r0.txt
<< A5231.r0.txt
<< K8000.r0.txt
<< K8001.r0.txt
<< K8010.r0.txt

:= $CHOOS$ ...
... $[\lambda t_\tau.[\lambda x_t.[\lambda y_t.[\lambda p_o.(\iota_{t(ot)}[\lambda q_t.(\vee_{ooo}(\wedge_{ooo}p_o(=_{ott}x_tq_t))(\wedge_{ooo}(\sim_{oo}p_o)(=_{ott}y_tq_t)))_o])_t]_{(to)}]_{(tot)}]_{(tott)}]$
wff 2080 : $[\lambda t.[\lambda x.[\lambda y.[\lambda p.(\iota\,[\lambda q.(\vee\,(\wedge\,p\,(=x\,q))\,(\wedge\,(\sim p)\,(=y\,q)))])]]]]_{\backslash 4o\backslash 3\backslash 2\tau}$:= $CHOOS$

##
Proof

##

.1

§= $_{t_\tau}$ $CHOOS_{\backslash 4o\backslash 3\backslash 2\tau} t_\tau x_t y_t T_o$
$= (CHOOS\, t\, x\, y\, T)\, (CHOOS\, t\, x\, y\, T)$
§\ $CHOOS_{\backslash 4o\backslash 3\backslash 2\tau} t_\tau$
$= (CHOOS\, t)\, [\lambda x.[\lambda y.[\lambda p.(\iota\, [\lambda q.(\vee\, (\wedge\, p\, (=x\, q))\, (\wedge\, (\sim p)\, (=y\, q)))])]]]$
§s %1 24 %0
$= (CHOOS\, t\, x\, y\, T)\, ([\lambda x.[\lambda y.[\lambda p.(\iota\, [\lambda q.(\vee\, (\wedge\, p\, (=x\, q))\, (\wedge\, (\sim p)\, (=y\, q)))])]]]\, x\, y\, T)$
§\ $[\lambda x_t.[\lambda y_t.[\lambda p_o.(\iota_{t(ot)}[\lambda q_t.(\vee_{ooo}(\wedge_{ooo} p_o(=_{ott} x_t q_t))(\wedge_{ooo}(\sim_{oo} p_o)(=_{ott} y_t q_t)))_o])_t]_{(to)}]_{(tot)}]x_t$
$= ([\lambda x.[\lambda y.[\lambda p.(\iota\, [\lambda q.(\vee\, (\wedge\, p\, (=x\, q))\, (\wedge\, (\sim p)\, (=y\, q)))])]]]\, x)\ \ldots$
$\ldots [\lambda y.[\lambda p.(\iota\, [\lambda q.(\vee\, (\wedge\, p\, (=x\, q))\, (\wedge\, (\sim p)\, (=y\, q)))])]]$
§s %1 12 %0
$= (CHOOS\, t\, x\, y\, T)\, ([\lambda y.[\lambda p.(\iota\, [\lambda q.(\vee\, (\wedge\, p\, (=x\, q))\, (\wedge\, (\sim p)\, (=y\, q)))])]]\, y\, T)$
§\ $[\lambda y_t.[\lambda p_o.(\iota_{t(ot)}[\lambda q_t.(\vee_{ooo}(\wedge_{ooo} p_o(=_{ott} x_t q_t))(\wedge_{ooo}(\sim_{oo} p_o)(=_{ott} y_t q_t)))_o])_t]_{(to)}]y_t$
$= ([\lambda y.[\lambda p.(\iota\, [\lambda q.(\vee\, (\wedge\, p\, (=x\, q))\, (\wedge\, (\sim p)\, (=y\, q)))])]]\, y)\ \ldots$
$\ldots [\lambda p.(\iota\, [\lambda q.(\vee\, (\wedge\, p\, (=x\, q))\, (\wedge\, (\sim p)\, (=y\, q)))])]$
§s %1 6 %0
$= (CHOOS\, t\, x\, y\, T)\, ([\lambda p.(\iota\, [\lambda q.(\vee\, (\wedge\, p\, (=x\, q))\, (\wedge\, (\sim p)\, (=y\, q)))])]\, T)$
§\ $[\lambda p_o.(\iota_{t(ot)}[\lambda q_t.(\vee_{ooo}(\wedge_{ooo} p_o(=_{ott} x_t q_t))(\wedge_{ooo}(\sim_{oo} p_o)(=_{ott} y_t q_t)))_o])_t]T_o$
$= ([\lambda p.(\iota\, [\lambda q.(\vee\, (\wedge\, p\, (=x\, q))\, (\wedge\, (\sim p)\, (=y\, q)))])]\, T)\ \ldots$
$\ldots (\iota\, [\lambda q.(\vee\, (\wedge\, T\, (=x\, q))\, (\wedge\, (\sim T)\, (=y\, q)))])$
§s %1 3 %0
$= (CHOOS\, t\, x\, y\, T)\, (\iota\, [\lambda q.(\vee\, (\wedge\, T\, (=x\, q))\, (\wedge\, (\sim T)\, (=y\, q)))])$
:= $LTMP5313 %0
wff 2115 : $= (CHOOS\, t\, x\, y\, T)\, (\iota\, [\lambda q.(\vee\, (\wedge\, T\, (=x\, q))\, (\wedge\, (\sim T)\, (=y\, q)))])_o$:= $LTMP5313

.2

§= $_o$ /15
$= (\vee\, (\wedge\, T\, (=x\, q))\, (\wedge\, (\sim T)\, (=y\, q)))\, (\vee\, (\wedge\, T\, (=x\, q))\, (\wedge\, (\sim T)\, (=y\, q)))$

%A5231a
$= (\sim T)\, F$:= A5231a
$=_{ooo}(\sim_{oo} T_o) F_o$:= A5231a
§s %1 29 %0
$= (\vee\, (\wedge\, T\, (=x\, q))\, (\wedge\, (\sim T)\, (=y\, q)))\, (\vee\, (\wedge\, T\, (=x\, q))\, (\wedge\, F\, (=y\, q)))$
:= $TMP5313 %0
wff 2120 : $= (\vee\, (\wedge\, T\, (=x\, q))\, (\wedge\, (\sim T)\, (=y\, q)))\, (\vee\, (\wedge\, T\, (=x\, q))\, (\wedge\, F\, (=y\, q)))_o$:= $TMP5313

%K8001b
$= (\wedge\, F\, x)\, F$:= K8001b
$=_{ooo}(\wedge_{ooo} F_o x_o) F_o$:= K8001b

use Proof Template A5221 (Sub): B → B [x/A]
:= $B5221 %0

wff 1805 : $=(\wedge F\, x)\, F_{o,\ldots}$:= $\$B5221$ $K8001b$
:= $\$T5221$ o
wff 2 : o_τ := $\$T5221$
:= $\$X5221$ x_o
wff 16 : x_o := $\$X5221$
:= $\$A5221$ %1/15
wff 2069 : $=y\, q_o$:= $\$A5221$
<< A5221.r0t.txt
:= $\$B5221$
:= $\$T5221$
:= $\$X5221$
:= $\$A5221$
%0
$=(\wedge F\, (=y\, q))\, F$
$=_{ooo}(\wedge_{ooo}F_o(=_{ott}y_t q_t))F_o$

%$\$TMP5313$
$=(\vee\, (\wedge T\, (=x\, q))\, (\wedge\, (\sim T)\, (=y\, q)))\, (\vee\, (\wedge T\, (=x\, q))\, (\wedge F\, (=y\, q)))$:= $\$TMP5313$
$=_{ooo}(\vee_{ooo}(\wedge_{ooo}T_o(=_{ott}x_t q_t))(\wedge_{ooo}(\sim_{oo}T_o)(=_{ott}y_t q_t)))\ldots$
$\ldots(\vee_{ooo}(\wedge_{ooo}T_o(=_{ott}x_t q_t))(\wedge_{ooo}F_o(=_{ott}y_t q_t)))$:= $\$TMP5313$
:= $\$TMP5313$
§s %0 7 %1
$=(\vee\, (\wedge T\, (=x\, q))\, (\wedge\, (\sim T)\, (=y\, q)))\, (\vee\, (\wedge T\, (=x\, q))\, F)$
:= $\$TMP5313$ %0
wff 2155 : $=(\vee\, (\wedge T\, (=x\, q))\, (\wedge\, (\sim T)\, (=y\, q)))\, (\vee\, (\wedge T\, (=x\, q))\, F)_o$:= $\$TMP5313$

%$K8000b$
$=(\wedge T\, x)\, x$:= $K8000b$
$=_{ooo}(\wedge_{ooo}T_o x_o)x_o$:= $K8000b$

use Proof Template A5221 (Sub): B → B [x/A]
:= $\$B5221$ %0
wff 594 : $=(\wedge T\, x)\, x_{o,\ldots}$:= $\$B5221$ $K8000b$
:= $\$T5221$ o
wff 2 : o_τ := $\$T5221$
:= $\$X5221$ x_o
wff 16 : x_o := $\$X5221$
:= $\$A5221$ %1/43
wff 2064 : $=x\, q_o$:= $\$A5221$
<< A5221.r0t.txt
:= $\$B5221$
:= $\$T5221$
:= $\$X5221$
:= $\$A5221$
%0
$=(\wedge T\, (=x\, q))\, (=x\, q)$
$=_{ooo}(\wedge_{ooo}T_o(=_{ott}x_t q_t))(=_{ott}x_t q_t)$

%$TMP5313
$= (\vee (\wedge T (= x\, q))\, (\wedge (\sim T)\, (= y\, q)))\, (\vee (\wedge T (= x\, q))\, F)$:= $TMP5313
$=_{ooo}(\vee_{ooo}(\wedge_{ooo}T_o(=_{ott}x_tq_t))(\wedge_{ooo}(\sim_{oo}T_o)(=_{ott}y_tq_t)))(\vee_{ooo}(\wedge_{ooo}T_o(=_{ott}x_tq_t))F_o)$
:= $TMP5313
:= $TMP5313
§s %0 13 %1
$= (\vee (\wedge T (= x\, q))\, (\wedge (\sim T)\, (= y\, q)))\, (\vee (= x\, q)\, F)$
:= $TMP5313 %0
wff 2191 : $= (\vee (\wedge T (= x\, q))\, (\wedge (\sim T)\, (= y\, q)))\, (\vee (= x\, q)\, F)_o$:= $TMP5313

%$K8010a$
$= (\vee x\, F)\, x$:= $K8010a$
$=_{ooo}(\vee_{ooo}x_oF_o)x_o$:= $K8010a$

use Proof Template A5221 (Sub): B → B [x/A]
:= $B5221 %0
wff 2028 : $= (\vee x\, F)\, x_o$:= $B5221 $K8010a$
:= $T5221 o
wff 2 : o_τ := $T5221
:= $X5221 x_o
wff 16 : x_o := $X5221
:= $A5221 %1/13
wff 2064 : $= x\, q_{o,\dots}$:= $A5221
<< A5221.r0t.txt
:= $B5221
:= $T5221
:= $X5221
:= $A5221
%0
$= (\vee (= x\, q)\, F)\, (= x\, q)$
$=_{ooo}(\vee_{ooo}(=_{ott}x_tq_t)F_o)(=_{ott}x_tq_t)$

%$TMP5313
$= (\vee (\wedge T (= x\, q))\, (\wedge (\sim T)\, (= y\, q)))\, (\vee (= x\, q)\, F)$:= $TMP5313
$=_{ooo}(\vee_{ooo}(\wedge_{ooo}T_o(=_{ott}x_tq_t))(\wedge_{ooo}(\sim_{oo}T_o)(=_{ott}y_tq_t)))(\vee_{ooo}(=_{ott}x_tq_t)F_o)$:=
$TMP5313
:= $TMP5313
§s %0 3 %1
$= (\vee (\wedge T (= x\, q))\, (\wedge (\sim T)\, (= y\, q)))\, (= x\, q)$

.3

%$LTMP5313
$= (CHOOS\, t\, x\, y\, T)\, (\iota\, [\lambda q.(\vee (\wedge T (= x\, q))\, (\wedge (\sim T)\, (= y\, q))))])$:= $LTMP5313
$=_{ott}(CHOOS_{\backslash 4o\backslash 3\backslash 2\tau}t_\tau x_t y_t T_o)\dots$
$\dots(\iota_{t(ot)}[\lambda q_t.(\vee_{ooo}(\wedge_{ooo}T_o(=_{ott}x_tq_t))(\wedge_{ooo}(\sim_{oo}T_o)(=_{ott}y_tq_t)))_o])$:= $LTMP5313
:= $LTMP5313
§s %0 15 %1
$= (CHOOS\, t\, x\, y\, T)\, (\iota\, [\lambda q.(= x\, q)])$

:= $TMP5313 %0
wff 2230 : $=(CHOOS\,t\,x\,y\,T)\,(\iota\,[\lambda q.(=x\,q)])_o$:= $TMP5313

.4

use Proof Template: A5205 Substitutions
:= $AA5205 o
wff 2 : o_τ := $AA5205
:= $BA5205 t_τ
wff 4 : t_τ := $BA5205
:= $FA5205 $=_{o\,\$BA5205\,\$BA5205}x_{\$BA5205}$
wff 115 : $=x_{o\,\$BA5205}$:= $FA5205
<< a5205_substitutions.r0t.txt
:= $AA5205
:= $BA5205
:= $FA5205
%0
$=(=x)\,[\lambda y.(=x\,y)]$
$=_{o(ot)(ot)}(=_{ott}x_t)[\lambda y_t.(=_{ott}x_t y_t)_o]$

§r /3 q_t
$=[\lambda y.(=x\,y)]\,[\lambda q.(=x\,q)]$
§s %1 3 %0
$=(=x)\,[\lambda q.(=x\,q)]$

use Proof Template A5201b (Swap): A = B → B = A
<< A5201b.r0t.txt
%0
$=[\lambda q.(=x\,q)]\,(=x)$
$=_{o(ot)(ot)}[\lambda q_t.(=_{ott}x_t q_t)_o](=_{ott}x_t)$

%$TMP5313
$=(CHOOS\,t\,x\,y\,T)\,(\iota\,[\lambda q.(=x\,q)])$:= $TMP5313
$=_{ott}(CHOOS_{\backslash 4o\backslash 3\backslash 2\tau}t_\tau x_t y_t T_o)(\iota_{t(ot)}[\lambda q_t.(=_{ott}x_t q_t)_o])$:= $TMP5313
:= $TMP5313
§s %0 7 %1
$=(CHOOS\,t\,x\,y\,T)\,(\iota\,(=x))$
:= $TMP5313 %0
wff 2376 : $=(CHOOS\,t\,x\,y\,T)\,(\iota\,(=x))_o$:= $TMP5313

.5

%A5
$=(\iota\,(=y))\,y$:= $A5$
$=_{ott}(\iota_{t(ot)}(=_{ott}y_t))y_t$:= $A5$

use Proof Template A5221 (Sub): B → B [x/A]
:= $B5221 %0
wff 207 : $=(\iota\,(=y))\,y_o$:= $B5221 $A5$

:= $T5221 t_τ
wff 4 : t_τ := $T5221
:= $X5221 $y_{\$T5221}$
wff 105 : $y_{\$T5221}$:= $X5221
:= $A5221 $x_{\$T5221}$
wff 24 : $x_{\$T5221}$:= $A5221
<< A5221.r0t.txt
:= $B5221
:= $T5221
:= $X5221
:= $A5221
%0
$= (\iota\,(= x))\,x$
$=_{ott}(\iota_{t(ot)}(=_{ott}x_t))x_t$

%$TMP5313
$= (CHOOS\,t\,x\,y\,T)\,(\iota\,(= x))$:= $TMP5313
$=_{ott}(CHOOS_{\backslash 4o\backslash 3\backslash 2\tau}t_\tau x_t y_t T_o)(\iota_{t(ot)}(=_{ott}x_t))$:= $TMP5313
:= $TMP5313
§s %0 3 %1
$= (CHOOS\,t\,x\,y\,T)\,x$
:= $LTMP5313 %0
wff 2424 : $= (CHOOS\,t\,x\,y\,T)\,x_o$:= $LTMP5313

.6

§= $_{t_\tau}$ $CHOOS_{\backslash 4o\backslash 3\backslash 2\tau}t_\tau x_t y_t F_o$
$= (CHOOS\,t\,x\,y\,F)\,(CHOOS\,t\,x\,y\,F)$
§\ $CHOOS_{\backslash 4o\backslash 3\backslash 2\tau}t_\tau$
$= (CHOOS\,t)\,[\lambda x.[\lambda y.[\lambda p.(\iota\,[\lambda q.(\vee\,(\wedge\,p\,(= x\,q))\,(\wedge\,(\sim p)\,(= y\,q)))])]]]$
§s %1 24 %0
$= (CHOOS\,t\,x\,y\,F)\,([\lambda x.[\lambda y.[\lambda p.(\iota\,[\lambda q.(\vee\,(\wedge\,p\,(= x\,q))\,(\wedge\,(\sim p)\,(= y\,q)))])]]]\,x\,y\,F)$
§\ $[\lambda x_t.[\lambda y_t.[\lambda p_o.(\iota_{t(ot)}[\lambda q_t.(\vee_{ooo}(\wedge_{ooo}p_o(=_{ott}x_t q_t))(\wedge_{ooo}(\sim_{oo}p_o)(=_{ott}y_t q_t)))_o])_t]_{(to)}]_{(tot)}]x_t$
$= ([\lambda x.[\lambda y.[\lambda p.(\iota\,[\lambda q.(\vee\,(\wedge\,p\,(= x\,q))\,(\wedge\,(\sim p)\,(= y\,q)))])]]]\,x)\,\ldots$
$\ldots\,[\lambda y.[\lambda p.(\iota\,[\lambda q.(\vee\,(\wedge\,p\,(= x\,q))\,(\wedge\,(\sim p)\,(= y\,q)))])]]$
§s %1 12 %0
$= (CHOOS\,t\,x\,y\,F)\,([\lambda y.[\lambda p.(\iota\,[\lambda q.(\vee\,(\wedge\,p\,(= x\,q))\,(\wedge\,(\sim p)\,(= y\,q)))])]]\,y\,F)$
§\ $[\lambda y_t.[\lambda p_o.(\iota_{t(ot)}[\lambda q_t.(\vee_{ooo}(\wedge_{ooo}p_o(=_{ott}x_t q_t))(\wedge_{ooo}(\sim_{oo}p_o)(=_{ott}y_t q_t)))_o])_t]_{(to)}]y_t$
$= ([\lambda y.[\lambda p.(\iota\,[\lambda q.(\vee\,(\wedge\,p\,(= x\,q))\,(\wedge\,(\sim p)\,(= y\,q)))])]]\,y)\,\ldots$
$\ldots\,[\lambda p.(\iota\,[\lambda q.(\vee\,(\wedge\,p\,(= x\,q))\,(\wedge\,(\sim p)\,(= y\,q)))])]$
§s %1 6 %0
$= (CHOOS\,t\,x\,y\,F)\,([\lambda p.(\iota\,[\lambda q.(\vee\,(\wedge\,p\,(= x\,q))\,(\wedge\,(\sim p)\,(= y\,q)))])]\,F)$
§\ $[\lambda p_o.(\iota_{t(ot)}[\lambda q_t.(\vee_{ooo}(\wedge_{ooo}p_o(=_{ott}x_t q_t))(\wedge_{ooo}(\sim_{oo}p_o)(=_{ott}y_t q_t)))_o])_t]F_o$
$= ([\lambda p.(\iota\,[\lambda q.(\vee\,(\wedge\,p\,(= x\,q))\,(\wedge\,(\sim p)\,(= y\,q)))])]\,F)\,\ldots$
$\ldots\,(\iota\,[\lambda q.(\vee\,(\wedge\,F\,(= x\,q))\,(\wedge\,(\sim F)\,(= y\,q)))])$
§s %1 3 %0
$= (CHOOS\,t\,x\,y\,F)\,(\iota\,[\lambda q.(\vee\,(\wedge\,F\,(= x\,q))\,(\wedge\,(\sim F)\,(= y\,q)))])$

%$A5231b$

```
#                    =(∼F)T          :=   A5231b
#                    =ooo(∼ooFo)To          :=   A5231b
§s %1 125 %0
#                    =(CHOOS t x y F)(ι[λq.(∨(∧F(=x q))(∧T(=y q))))])
:= $TMP5313 %0
# wff 2447 : =(CHOOS t x y F)(ι[λq.(∨(∧F(=x q))(∧T(=y q))))])o := $TMP5313

%K8001b
#                    =(∧F x)F          :=   K8001b
#                    =ooo(∧oooFoxo)Fo          :=   K8001b

## use Proof Template A5221 (Sub):   B   →   B [x/A]
:= $B5221 %0
# wff 1805 : =(∧F x)F_{o,...}          :=   $B5221   K8001b
:= $T5221 o
# wff 2 : oτ          :=   $T5221
:= $X5221 xo
# wff 16 : xo          :=   $X5221
:= $A5221 %1/123
# wff 2064 : =x q_{o,...}          :=   $A5221
<< A5221.r0t.txt
:= $B5221
:= $T5221
:= $X5221
:= $A5221
%0
#                    =(∧F(=x q))F
#                    =ooo(∧oooFo(=ottxtqt))Fo

%$TMP5313
#                    =(CHOOS t x y F)(ι[λq.(∨(∧F(=x q))(∧T(=y q))))])          :=   $TMP5313
#                    =ott(CHOOS\4o\3\2τtτxtytFo)...
...(ιt(ot)[λqt.(∨ooo(∧oooFo(=ottxtqt))(∧oooTo(=ottytqt)))o])          :=   $TMP5313
:= $TMP5313
§s %0 61 %1
#                    =(CHOOS t x y F)(ι[λq.(∨F(∧T(=y q))))])
:= $TMP5313 %0
# wff 2459 : =(CHOOS t x y F)(ι[λq.(∨F(∧T(=y q))))])o          :=   $TMP5313

%K8000b
#                    =(∧T x)x          :=   K8000b
#                    =ooo(∧oooTo xo)xo          :=   K8000b

## use Proof Template A5221 (Sub):   B   →   B [x/A]
:= $B5221 %0
# wff 594 : =(∧T x)x_{o,...}          :=   $B5221   K8000b
:= $T5221 o
# wff 2 : oτ          :=   $T5221
```

:= $X5221 x_o
wff 16 : x_o := $X5221
:= $A5221 %1/63
wff 2069 : $=y\,q_o$:= $A5221
<< A5221.r0t.txt
:= $B5221
:= $T5221
:= $X5221
:= $A5221
%0
$=(\wedge T\,(=y\,q))\,(=y\,q)$
$=_{ooo}(\wedge_{ooo}T_o(=_{ott}y_tq_t))(=_{ott}y_tq_t)$

%$TMP5313
$=(CHOOS\,t\,x\,y\,F)\,(\iota\,[\lambda q.(\vee\,F\,(\wedge\,T\,(=y\,q)))])$:= $TMP5313
$=_{ott}(CHOOS_{\backslash 4o\backslash 3\backslash 2\tau}t_\tau x_t y_t F_o)(\iota_{t(ot)}[\lambda q_t.(\vee_{ooo}F_o(\wedge_{ooo}T_o(=_{ott}y_tq_t)))_o])$:=
$TMP5313
:= $TMP5313
§s %0 31 %1
$=(CHOOS\,t\,x\,y\,F)\,(\iota\,[\lambda q.(\vee\,F\,(=y\,q))])$
:= $TMP5313 %0
wff 2471 : $=(CHOOS\,t\,x\,y\,F)\,(\iota\,[\lambda q.(\vee\,F\,(=y\,q))])_o$:= $TMP5313

%$K8010b$
$=(\vee\,F\,x)\,x$:= $K8010b$
$=_{ooo}(\vee_{ooo}F_ox_o)x_o$:= $K8010b$

use Proof Template A5221 (Sub): B → B [x/A]
:= $B5221 %0
wff 2058 : $=(\vee\,F\,x)\,x_o$:= $B5221 $K8010b$
:= $T5221 o
wff 2 : o_τ := $T5221
:= $X5221 x_o
wff 16 : x_o := $X5221
:= $A5221 %1/31
wff 2069 : $=y\,q_{o,\dots}$:= $A5221
<< A5221.r0t.txt
:= $B5221
:= $T5221
:= $X5221
:= $A5221
%0
$=(\vee\,F\,(=y\,q))\,(=y\,q)$
$=_{ooo}(\vee_{ooo}F_o(=_{ott}y_tq_t))(=_{ott}y_tq_t)$

%$TMP5313
$=(CHOOS\,t\,x\,y\,F)\,(\iota\,[\lambda q.(\vee\,F\,(=y\,q))])$:= $TMP5313
$=_{ott}(CHOOS_{\backslash 4o\backslash 3\backslash 2\tau}t_\tau x_t y_t F_o)(\iota_{t(ot)}[\lambda q_t.(\vee_{ooo}F_o(=_{ott}y_tq_t))_o])$:= $TMP5313
:= $TMP5313

§s %0 15 %1
$= (CHOOS\, t\, x\, y\, F)\, (\iota\, [\lambda q.(= y\, q)])$
:= $TMP5313$ %0
wff 2509 : $= (CHOOS\, t\, x\, y\, F)\, (\iota\, [\lambda q.(= y\, q)])_o$:= $TMP5313$

.7

use Proof Template: A5205 Substitutions
:= $AA5205$ o
wff 2 : o_τ := $AA5205$
:= $BA5205$ t_τ
wff 4 : t_τ := $BA5205$
:= $FA5205$ $=_{o\,\$BA5205\,\$BA5205 z_{\$BA5205}}$
wff 2510 : $= z_{o\,\$BA5205}$:= $FA5205$
<< a5205_substitutions.r0t.txt
:= $AA5205$
:= $BA5205$
:= $FA5205$
%0
$= (= z)\, [\lambda y.(= z\, y)]$
$=_{o(ot)(ot)} (=_{ott} z_t) [\lambda y_t.(=_{ott} z_t y_t)_o]$

§r /3 q_t
$= [\lambda y.(= z\, y)]\, [\lambda q.(= z\, q)]$
§s %1 3 %0
$= (= z)\, [\lambda q.(= z\, q)]$

use Proof Template A5221 (Sub): B → B [x/A]
:= $B5221$ %0
wff 2529 : $= (= z)\, [\lambda q.(= z\, q)]_o$:= $B5221$
:= $T5221$ t_τ
wff 4 : t_τ := $T5221$
:= $X5221$ $z_{\$T5221}$
wff 83 : $z_{\$T5221}$:= $X5221$
:= $A5221$ $y_{\$T5221}$
wff 105 : $y_{\$T5221}$:= $A5221$
<< A5221.r0t.txt
:= $B5221$
:= $T5221$
:= $X5221$
:= $A5221$
%0
$= (= y)\, [\lambda q.(= y\, q)]$
$=_{o(ot)(ot)} (=_{ott} y_t) [\lambda q_t.(=_{ott} y_t q_t)_o]$

use Proof Template A5201b (Swap): A = B → B = A
<< A5201b.r0t.txt
%0
$= [\lambda q.(= y\, q)]\, (= y)$

$=_{o(ot)(ot)}[\lambda q_t.(=_{ott} y_t q_t)_o](=_{ott} y_t)$

%$\$TMP5313$
$=(CHOOS\, t\, x\, y\, F)\,(\iota\,[\lambda q.(=y\, q)])$:= $\$TMP5313$
$=_{ott}(CHOOS_{\backslash 4o\backslash 3\backslash 2\tau} t_\tau x_t y_t F_o)(\iota_{t(ot)}[\lambda q_t.(=_{ott} y_t q_t)_o])$:= $\$TMP5313$
:= $\$TMP5313$
§s %0 7 %1
$=(CHOOS\, t\, x\, y\, F)\,(\iota\,(=y))$

%$A5$
$=(\iota\,(=y))\, y$:= $A5$
$=_{ott}(\iota_{t(ot)}(=_{ott} y_t))y_t$:= $A5$
§s %1 3 %0
$=(CHOOS\, t\, x\, y\, F)\, y$

.8

use Proof Template K8020: A, B → A ∧ B
:= $\$A8020$ $=_{ott}(CHOOS_{\backslash 4o\backslash 3\backslash 2\tau} t_\tau x_t y_t T_o) x_t$
wff 2424 : $=(CHOOS\, t\, x\, y\, T)\, x_o$:= $\$A8020$ $\$LTMP5313$
:= $\$LTMP5313$
:= $\$B8020$ %0
wff 2579 : $=(CHOOS\, t\, x\, y\, F)\, y_o$:= $\$B8020$
<< K8020.r0t.txt
:= $\$A8020$
:= $\$B8020$

:= $A5313$ %0
wff 2614 : $\wedge(=(CHOOS\, t\, x\, y\, T)\, x)\,(=(CHOOS\, t\, x\, y\, F)\, y)_o$:= $A5313$

##
Q.E.D.
##

%0
$\wedge(=(CHOOS\, t\, x\, y\, T)\, x)\,(=(CHOOS\, t\, x\, y\, F)\, y)$:= $A5313$
$\wedge_{ooo}(=_{ott}(CHOOS_{\backslash 4o\backslash 3\backslash 2\tau} t_\tau x_t y_t T_o) x_t)(=_{ott}(CHOOS_{\backslash 4o\backslash 3\backslash 2\tau} t_\tau x_t y_t F_o) y_t)$:=
$A5313$

5.1.45 Results for File A6100.r0.txt

##
Proof A6100: Peano's Postulate No. 1 for Andrews' Definition of Natural Numbers
##
##
Source: [Andrews 2002 (ISBN 1-4020-0763-9), p. 261]
##

:= $E5209
:= $T5209
:= $X5209
:= $A5209
%0
$= (= \$FA3\,\$GA3)\,(\forall\,\$BA3\,[\lambda x.(= (\$FA3\,x)\,(\$GA3\,x))])$
$=_{ooo}(=_{o(\$AA3\,\$BA3)(\$AA3\,\$BA3)}\$FA3_{\$AA3\,\$BA3}\$GA3_{\$AA3\,\$BA3})\ldots$
$\ldots(\forall_{o(o\backslash 3)\tau}\$BA3_{\tau}[\lambda x_{\$BA3}.(=_{o\,\$AA3\,\$AA3}(\$FA3_{\$AA3\,\$BA3}x_{\$BA3})(\$GA3_{\$AA3\,\$BA3}x_{\$BA3}))_o])$
Include end (axiom3_substitutions.r0t.txt) [newfile=(axiom3_substitutions.r0.txt)]
>>>

##
Undefine Syntactical Variables
##

:= $AA3
:= $BA3
:= $FA3
:= $GA3

##
Q.E.D.
##

%0
$= (= y\,z)\,(\forall\,u\,[\lambda x.(= (y\,x)\,(z\,x))])$
$=_{ooo}(=_{o(tu)(tu)}y_{tu}z_{tu})(\forall_{o(o\backslash 3)\tau}u_{\tau}[\lambda x_u.(=_{ott}(y_{tu}x_u)(z_{tu}x_u))_o])$

5.1.86 Results for File axioms.r0.txt

##
Axioms
##
##
Source: [Andrews 2002 (ISBN 1-4020-0763-9), p. 213]
##

Written by Ken Kubota (<mail@kenkubota.de>).
##
This file is part of the work “On the Theory of Mathematical Forms”.
For more information visit: <http://dx.doi.org/10.4444/100.10>
##

<< definitions1.r0.txt

##

```
##   Axiom 1: Truth and Falsehood are the only truth values
##

:=   A1  =ooo(∧ooo(goo To)(goo Fo))(∀o(o\3)τ oτ[λxo.(goo xo)o])
# wff      90   :        = (∧ (g T) (g F)) (∀ o [λx.(g x)])o        :=   A1
§!  A1
#                  = (∧ (g T) (g F)) (∀ o [λx.(g x)])        :=   A1


##
##   Axiom 2: One of the Basic Properties of Equality
##

:=   A2  ⊃ooo(=oaa xa ya)(=ooo(hoa xa)(hoa ya))
# wff      104   :        ⊃ (= x y) (= (h x) (h y))o        :=   A2
§!  A2
#                  ⊃ (= x y) (= (h x) (h y))        :=   A2


##
##   Axiom 3: Axiom of Extensionality
##

:=   A3  =ooo(=o(ab)(ab) fab gab)(∀o(o\3)τ bτ[λxb.(=oaa(fab xb)(gab xb))o])
# wff      124   :        = (= f g) (∀ b [λx.(= (f x) (g x))])o        :=   A3
§!  A3
#                  = (= f g) (∀ b [λx.(= (f x) (g x))])        :=   A3


##
##   Axiom 4: Axiom of Lambda Conversion
##

## Replaced by Rule 2 (Lambda Conversion)
## [cf. Andrews 2002 (ISBN 1-4020-0763-9), p. 218 f. (5207)]
##
## "5207 could be taken as an axiom schema in place of 4_1 - 4_5,
##   and for some purposes this would be desirable,
##   since 5207 has a conceptual simplicity and unity
##   which is not apparent in 4_1 - 4_5." [Andrews 2002, p. 214]


##
##   Axiom 5: Axiom of Descriptions
##

:=   A5  =ott(ιt(ot)(=ott yt))yt
# wff      129   :        = (ι (= y)) yo        :=   A5
§!  A5
```

$= (\iota\,(= y))\,y$:= $A5$

5.1.87 Results for File basics.r0.txt

```
##
##   Basics
##
##
##   Source: [Kubota 2015 (doi: 10.4444/100.10)]
##
##   Copyright (c) 2015 Owl of Minerva Press GmbH. All rights reserved.
##   Written by Ken Kubota (<mail@kenkubota.de>).
##
##   This file is part of the work "On the Theory of Mathematical Forms".
##   For more information visit: <http://dx.doi.org/10.4444/100.10>
##

<< definitions1.r0.txt
<< definitions2.r0.txt
<< definitions3.r0.txt
<< axioms.r0.txt
```

5.1.88 Results for File composition.r0.txt

```
##
##   Associativity of the Composition of Functions
##
##
##   Source: [Kubota 2015 (doi: 10.4444/100.10)]
##
##   Copyright (c) 2015 Owl of Minerva Press GmbH. All rights reserved.
##   Written by Ken Kubota (<mail@kenkubota.de>).
##
##   This file is part of the work "On the Theory of Mathematical Forms".
##   For more information visit: <http://dx.doi.org/10.4444/100.10>
##

<< basics.r0.txt
```

:= $COMPS$ ...
... $[\lambda a_\tau.[\lambda b_\tau.[\lambda c_\tau.[\lambda g_{ab}.[\lambda f_{bc}.[\lambda x_c.(g_{ab}(f_{bc}x_c))_a]_{(ac)}]_{(ac(bc))}]_{(ac(bc)(ab))}]_{(a\backslash 4(b\backslash 4)(ab)\tau)}]_{(a\backslash 4(\backslash 5\backslash 4)(a\backslash 4)\tau\tau)}]$
wff 233 : $[\lambda a.[\lambda b.[\lambda c.[\lambda g.[\lambda f.[\lambda x.(g\,(f\,x))]]]]]]_{\backslash 6\backslash 4(\backslash 5\backslash 4)(\backslash 5\backslash 4)\tau\tau\tau}$:= $COMPS$

.1

:= $\$GF$ $COMPS_{\backslash 6\backslash 4(\backslash 5\backslash 4)(\backslash 5\backslash 4)\tau\tau\tau} u_\tau v_\tau w_\tau g_{uv} f_{vw}$
wff 264 : $COMPS\,u\,v\,w\,g\,f_{uw}$:= $\$GF$